Universality and quantum effects in one-component critical fluids

Yves Garrabos*

*Equipe du Supercritique pour l'Environnement, les Matériaux et l'Espace - Institut de Chimie de la Matière Condensée de Bordeaux -
Centre National de la Recherche Scientifique - Université Bordeaux I - 87, avenue du Docteur Schweitzer,
F 33608 PESSAC Cedex France*

Nonuniversal scale transformations of the physical fields are extended to pure quantum fluids and used to calculate the susceptibility, the specific heat, and the order parameter density along the critical isochore of He^3 near its liquid-vapor critical point. Within the so-called preasymptotic domain, where the Wegner expansion restricted to the first term of confluent corrections to scaling is expected to be valid, the results are in agreement with the experimental measurements and recent predictions, either based on the minimal-substraction renormalization and the massive renormalization schemes within the $\Phi_{d=3}^4(n=1)$ model, or based on the crossover parametric equation of state for Ising-like systems.

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I. INTRODUCTION

It is well known that the thermodynamic quantities of real pure fluids close to their gas-liquid critical point (CP) follow the asymptotic power-law behavior predicted for the three-dimensional (3D) Ising-like universality class in the *asymptotic* critical domain where $\kappa \ll \Lambda_0$ [1]. The distance to the critical point is measured by the parameter κ, related to the inverse correlation length ξ^{-1}. ξ characterizes the spatial extent of the diverging fluctuations of the local density which is related to the order parameter density of the gas-liquid transition. Λ_0, the so-called "cutoff" wave number [1], is a (nonuniversal) finite wave number characterizing a discrete microscopic structure of a fluid with spacing Λ_0^{-1}. The critical thermophysical behavior of the fluid properties occurs when $\xi\Lambda_0 \gg 1$. Asymptotically close to the critical point, this microscopic parameter Λ_0 which characterizes each pure fluid, becomes unimportant when the thermodynamic properties become singular. As a result, all the pure fluids in their asymptotic critical domain obey to the two-scale factor universality associated to hyperscaling. Their properties can then be described by the same reduced equation of state (EOS) and the same correlation functions, using only two dimensionless parameters which are fluid dependent, in conformity with the two-scale-factor universality of the 3D Ising-like universality class.

However, it is also now well established that away from this asymptotic critical region, the properties of real pure fluids deviate from hyperscaling. This deviation leads to a crossover phenomenon which reflects a competition between universality and nonuniversality when $\xi\Lambda_0 \gtrsim 1$. This crossover problem has been investigated in considerable detail, mainly in the classical-to-critical crossover framework of field theory (FT) [2]. The resulting field theoretical crossover functions describe the crossover behavior of the $\phi_{d=3}^4(n=1)$ model in the Ising-like universality class in three dimensions ($n=1$ is the dimension of the order parameter density for the critical transition, and $d=3$ is the space dimension of the

system). A better understanding of nonuniversal behavior linked to finite values (although large) of the correlation length can then be accounted for by a restricted summation of the Wegner expansion [3], which introduces one additional system-dependent parameter to characterize the preasymptotic domain [4,5] (as discussed in Ref. [5] the singular power laws expressed at the first-order of the Wegner expansion are expected to be valid within the preasymptotic domain). However, the values of the adjustable parameters are then dependent on the approximations used in each particular renormalization scheme. As a practical result, the microscopic length, the crossover parameter, as well as the two asymptotic scale factors, form a larger set of adjustable parameters, including obviously an extended set of measurable critical point coordinates. Therefore, the exact nature of the two asymptotic scale factors for the fluid physical fields, still remains unknown. This is still the reason of a debate [6], due to the fact that fluid variables have no definite critical scaling dimensionality at finite distance to the critical point.

The asymptotic existence of such two scale factors for the one-component fluid subclass, was initially postulated in [7,8] on a phenomenological basis supporting the asymptotic results of the massive renormalization scheme [4,5,9,10] of the FT framework. It was hypothesized that the complete information to estimate asymptotic singular behavior of fluids is provided by the experimental critical point location, i.e., by a minimal parameter set, noted $Q_{c,a_{\bar{p}}}^{\min}$ [11], composed of four (generalized) critical coordinates [see Eq. (2) below] (the subscript c refers to a property defined at the critical point, while the subscript $\bar{p}$ refers to a property normalized per particle). This minimal set defines the critical point location on the equilibrium phase surface given by equation $\Phi_{a_{\bar{p}}}^p(p, v_{\bar{p}}, T) = 0$, where p is the pressure, $v_{\bar{p}} = \frac{V}{N}$ is the volume per particle, and T is the temperature (N is the total number of particles filling a total volume V). The generalized coordinates are composed of three usual critical point coordinates and one preferred direction [see Eq. (1) below] of the tangent plane to the phase surface. Using xenon as a standard critical fluid [7,8,10], it was then proposed to perform adequate scale dilatations of the two relevant physical variables for each one-component fluid. Applying such a scale dilatation

*Electronic address: garrabos@icmcb-bordeaux.cnrs.fr

method, the physical singular behavior of any one-component fluid could be renormalized on the corresponding "master" (i.e., unique) singular behavior, where master (i.e., constant amplitudes) features with respect to the one-component fluid subclass are in accordance with universal (i.e., universal amplitude combinations) features with respect to the complete 3D Ising-like universality class.

Specifically, this initial substantiation of master scaling is based on the explicit choice of the same metric factors for thermodynamic *and* correlation functions. That permits an unambiguous definition of the microscopic length Λ_0^{-1} proportional to a critical length scale factor $\alpha_c = \left(\frac{k_B T_c}{p_c}\right)^{1/d}$ made from an appropriate combination of the critical temperature T_c and pressure p_c coordinates [k_B is the Boltzmann constant and $d=3$]. From the well-known short range of the Lennard-Jones- (LJ-) like molecular interactions in one-component fluids [12], characterized by the equilibrium position r_e^{LJ} between two interacting particles, we have $\Lambda_0^{-1} = \alpha_c \cong 2r_e^{LJ}$ [7], ignoring the possible contribution of quantum effects on Λ_0^{-1}. In this paper, using the recent experimental measurements near the critical point of ^{3}He [13,14], we extend this scale dilatation method to quantum fluids. This extension is based on a phenomenological modification of the nonquantum renormalized critical behavior, which is only valid at the critical temperature. Since experimental values of the minimum critical set already contain their actual contribution of quantum effects, we expect that remaining part of quantum effects only affect the microscopic length $(\Lambda_0)^{-1}$, in such a relative way that $(\Lambda_0 \Lambda_{qe}^*)^{-1} = \alpha_c$ [see below Eq. (14)]. The adjustable dimensionless parameter Λ_{qe}^* is introduced here in order to maintain the master features observed for the one-component fluid subclass. Therefore, in addition to the minimal set of four critical parameters, the renormalized variables need to contain only one supplementary well-defined dimensionless parameter Λ_{qe}^*, whose value is, either fluid independent ($\Lambda_{qe}^* = 1$) in the absence of quantum effects, or quantum-fluid dependent ($\Lambda_{qe}^* > 1$) in the presence of quantum effects, without violating the asymptotic universal features of the 3D Ising-like universality class.

The paper is organized as follows. In Sec. II, we recall the basic elements of the scale dilatation method and we introduce its extension to account for quantum effects on the microscopic length scale. In Sec. III, we consider the fitting results [14] obtained by Zhong *et al.* for ^{3}He to discuss the estimated value of the adjustable parameter $\Lambda_{qe}^*(^3\text{He}) = 1.11009$. Before conclusions, Sec. IV gives a brief comparison with three crossover modeling of ^{3}He critical properties.

II. SCALE DILATATION OF THE FLUID PHYSICAL VARIABLES

A. The minimal set $Q_{c,a_{\bar{p}}}^{\min}$ of four scale factors

As stated in the Introduction, the basic idea [7,8] of the scale dilatation method relies on a simple thermodynamic assertion concerning the thermodynamic information provided by the critical point location on the fluid phase surface of equation $\Phi_{a_{\bar{p}}}^p(p, v_{\bar{p}}, T) = 0$ [11]. The minimum of informa-

tion needed to predict singular thermodynamic behavior of a pure fluid is then given by (1) the three critical coordinates T_c, p_c, and $v_{\bar{p},c}$ of the liquid-vapor critical point and (2) the two preferred directions which define the position of the tangent plane to the phase surface at the critical point (both needed in order to characterize the linearized asymptotic approach towards the critical point along two well-defined thermodynamic paths). One direction is common to all pure fluids (since $\left(\frac{\partial p}{\partial v_{\bar{p}}}\right)_{T=T_c} = \left(\frac{\partial T}{\partial v_{\bar{p}}}\right)_{p=p_c} = 0$), and only the second direction

$$\gamma_c' = \left(\frac{\partial p}{\partial T}\right)_{v_{\bar{p}} = v_{\bar{p},c}} = \left(\frac{dp_{\text{sat}}}{dT}\right)_{T=T_c} \quad (1)$$

is characteristic of each pure fluid. p_{sat} is the saturation pressure in the non homogeneous domain. We note that

$$Q_{c,a_{\bar{p}}}^{\min} = \{T_c, p_c, v_{\bar{p},c}, \gamma_c'\} \quad (2)$$

is this minimal set made of four critical parameters.

From thermodynamic principles, this topological information concerns all the incipient equilibrium states very close to the unstable single critical point. From these four coordinates, we can calculate unequivocally the following four fluid characteristic parameters,

$$(\beta_c)^{-1} = k_B T_c \sim [\text{energy}], \quad (3)$$

$$\alpha_c = \left(\frac{k_B T_c}{p_c}\right)^{\frac{1}{d}} \sim [\text{length}], \quad (4)$$

$$Z_c = \frac{p_c m_{\bar{p}}}{\rho_c k_B T_c}, \quad (5)$$

$$Y_c = \left(\gamma_c' \frac{T_c}{P_c}\right) - 1, \quad (6)$$

where $\rho_c = \left(\frac{N}{V}\right)_c m_{\bar{p}}$ is the critical density of the system made of particles of known individual mass $m_{\bar{p}}$. $(\beta_c)^{-1}$ of Eq. (3) fixes the energy unit at the macroscopic scale. α_c of Eq. (4) fixes the length unit at the macroscopic scale. Equations (3) and (4) are sufficient to make dimensionless all the thermodynamic and correlation functions of pure fluids [7,8]. Z_c in Eq. (5) is the critical compression factor. We then introduce the useful compression factor $Z = \frac{-J(T,V,\mu_{\bar{p}})}{k_B T}$ as the dimensionless opposite form of the total Grand potential $J(T, V, \mu_{\bar{p}}) = -p(T, \mu_{\bar{p}}) \times V$, expressed in terms of its three natural intensive variables T, V, and $\mu_{\bar{p}}$. For the total system, $\mu_{\bar{p}}$ is the chemical potential per particle, i.e., the intensive variable conjugated to N, independent of p and T, respectively (ρ and T are the two other independent intensive variables, conjugated to V and S, respectively). From the experimental phase surface of equation $\Phi_{a_{\bar{p}}}^p(p, v_{\bar{p}}, T) = 0$, it is easy to construct another practical phase surface of equation $\Phi(Z, \tilde{\rho}, T^*) = 0$ where $\tilde{\rho} = \rho/\rho_c$ and $T^* = T/T_c$ are the useful dimensionless density and temperature variables, respectively. In such a representation of the fluid equilibrium states, the characteristics numbers Z_c of Eq. (5) and $Y_c Z_c$ made of

the product between Eqs. (5) and (6), are given by

$$Z_c = -\left[\left(\frac{\partial Z}{\partial \tilde{\rho}}\right)_{\varpi}\right]_{\mathrm{CP}} = -\left[\left(\frac{\partial Z}{\partial \rho}\right)_{\mathrm{LVE}}\right]_{\mathrm{CP}}, \qquad (7)$$

$$Y_c Z_c = \left[\left(\frac{\partial Z}{\partial T^*}\right)_{\rho}\right]_{\mathrm{CP}}, \qquad (8)$$

where all the derivatives refer to their values at the critical point coordinates, while subscript ϖ means any isocline at constant (critical) value of one intensive variable ϖ among T, p, or $\mu_{\bar{p}}$, and subscript "LVE" means the liquid vapor equilibrium line. Therefore, the two caracteristic numbers Z_c and $Y_c Z_c$ are the two "preferred" critical directions [15] at the critical point of the phase surface, for the critical isotherm path and the critical isochore path, respectively.

From basic modeling of a binary effective interaction characterized by a minimum energy well depth $\varepsilon_m^{\mathrm{LJ}}$ at the pair equilibrium position r_e^{LJ} between two particles, we obtain, $(\beta_c)^{-1} \cong \varepsilon_m^{\mathrm{LJ}}$ and $\alpha_c \cong 2 \ r_e^{\mathrm{LJ}}$, where $\varepsilon_m^{\mathrm{LJ}}$ and r_e^{LJ} are natural units for energy and length at the microscopic scale, respectively. Here the subscript LJ stands for a short-ranged Lennard-Jones-like potential [12]. It follows that α_c is a measure of the mean extension range of the attractive dispersion forces and

$$v_{c,I} = (\alpha_c)^d \qquad (9)$$

is the critical volume of the microscopic critical interaction cell. In such a configuration, the inverse of the critical compression factor has a clear physical meaning since

$$\frac{1}{Z_c} = \frac{v_{c,I}}{v_{\bar{p},c}} = n_{c,I}^* \qquad (10)$$

is the number of fluid particles filling the interaction cell at criticality, i.e., for $T=T_c$, $n=n_c$, and then $\Lambda_0 \xi = \infty$ [$v_{\bar{p},c} = \left(\frac{V}{N}\right)_{\mathrm{PC}}$ is the critical volume per particle, and n (n_c) is the (critical) number density]. Using this result, it is easy to formulate dimensionless thermodynamics in terms of normalization per particle (subscript $\bar{p}$), or in terms of normalization per critical interaction cell (subscript I). As an immediate consequence of Eqs. (10) and (8), $\frac{1}{Z_c}$ and Y_c are the two numbers attached to the critical interaction cell.

The next step consists in postulating that the two numbers $\{Z_c; Y_c\}$ are the remaining pair of dimensionless characteristic parameters at the scale of the critical interaction volume, whatever the selected one-component fluid. In addition, it is assumed that Z_c is the characteristic factor of the scaling at the critical point and along the critical isotherm, while Y_c is the characteristic factor of the scaling along the critical isochore. Rewriting Eq. (2) as

$$Q_{c,a_{\bar{p}}}^{\min} = \{(\beta_c)^{-1}, \alpha_c, Z_c, Y_c\}_{\mathrm{CIC}} \qquad (11)$$

we can expect that the complete information is contained in the four scale factors which characterize the critical interaction cell (CIC). Then, as initially proposed in [7,8], the master singular behavior of the correlation functions at exact criticality and along the critical isochore permits one to link unequivocally their associated asymptotic amplitudes $\hat{D}$ and

ξ_0^+ [16], to Z_c and Y_c, respectively, providing simultaneously the hyperscaling [8].

B. Quantum effects on the scale dilatation of physical fluid variables

The scheme given in [8] also requires that the inverse microscopic wave number Λ_0^{-1} is proportional to the characteristic length scale α_c. Now, due to the short ranged molecular interaction in light pure fluids [17], the influence of quantum mechanical effects changes appreciably the shape of the Lennard-Jones-like potential, and also slightly increases the range of this interparticle potential [12]. This qualitative evidence was demonstrated by introducing an effective potential, which is then a temperature-dependent quantity [18–20]. The quantum effects increase as temperature decreases. However, due to the formal analogy with the FT renormalization scheme, our rescaling is basically defined for the critical asymptotic domain, i.e., only when $T \cong T_c$. Moreover, since the use of the actual critical parameters already includes quantum effects, the remaining additional quantum effect, for $T \cong T_c$, acts only through the relative modification of the microscopic length at T_c. In the absence of theoretical support for this modification, we propose to normalize its contribution with respect to the microscopic inverse wave number defined for nonquantum fluids.

This contribution is expected to be small and limited to a positive departure from unity. This value, noted λ_c, can then be included in the two main phenomenological characteristics of quantum particles:

(i) Their low mass and size, accounted for using proportionality to the ratio $\frac{\Lambda_{T,c}}{\alpha_c}$ between the critical thermal wavelength,

$$\Lambda_{T,c} = \frac{h_P}{(2\pi m_{\bar{p}} k_B T_c)^{1/2}}$$

(where h_P is the Planck constant), and the microscopic critical range α_c of the interaction;

(ii) Their statistics (such as bosons, fermions, etc.), accounted for by introducing a supplementary free parameter, noted $\lambda_{q,f}$.

Therefore, the quantum corrections can be characterized by the following dimensionless factor:

$$\Lambda_{qe}^* = 1 + \lambda_c \qquad (12)$$

with

$$\lambda_c = \lambda_{q,f} \frac{\Lambda_{T,c}}{\alpha_c}, \qquad (13)$$

where $\lambda_c \geq 0$ is then the measure of the relative modification of the shape and range of molecular interaction due to the quantum effects.

Since the quantum effects increase slightly the range of the molecular interaction, we postulate that the corrected microscopic wave number is given by

$$\Lambda_0 \Lambda_{qe}^* = \frac{1}{\alpha_c} \tag{14}$$

(in a nonquantum fluid, our previous implicit relation was $\Lambda_0 = \frac{1}{\alpha_c}$). We expect that the rescaled quantum-fluid correlation length ℓ_{qf}^* presents the master divergence previously defined for all the nonquantum one-component fluids [8,21]. Then,

$$\ell_{\text{qf}}^* = \Lambda_0 \xi = (\Lambda_{qe}^*)^{-1} \xi^* \tag{15}$$

with

$$\xi^* = \frac{\xi}{\alpha_c}. \tag{16}$$

Similarly, any rescaled singular thermodynamic property of the quantum fluid can be formulated from dimensional analysis, in order to account for its proper Λ_{qe}^* contribution within $v_{c,I}$, which maintains valid the previous master hypotheses made for the nonquantum fluid subclass.

Therefore, due to the formal analogy between the scale dilatation method [8] and the basic hypotheses of the renormalization group approach [22,23], all the above quantum corrections are intrinsically accounted for according to our renormalization scheme, provided that the transformations (dilatations) of the two relevant physical fields are made using the following analytical relations:

$$\mathcal{T}_{\text{qf}}^* \equiv \mathcal{T}^* = Y_c \Delta \tau^*, \tag{17}$$

$$\mathcal{H}_{\text{qf}}^* = (\Lambda_{qe}^*)^2 \mathcal{H}^* = (\Lambda_{qe}^*)^2 (Z_c)^{-d/2} \Delta h^*. \tag{18}$$

Consequently, the dilatation of the physical order parameter density reads

$$\mathcal{M}_{\text{qf}}^* = \Lambda_{qe}^* \mathcal{M}^* = \Lambda_{qe}^* (Z_c)^{d/2} \Delta m^*. \tag{19}$$

In Eqs. (17)–(19),

$$\Delta \tau^* = k_B \beta_c (T - T_c), \tag{20}$$

$$\Delta h^* = \beta_c (\mu_{\bar{p}} - \mu_{\bar{p},c}) \tag{21}$$

while

$$\Delta m^* = (n - n_c)(\alpha_c)^d. \tag{22}$$

$\mu_{\bar{p},c}$ is the critical chemical potential per particle. Obviously, in Eqs. (17)–(19), $\mathcal{T}^*$, $\mathcal{H}^*$, and $\mathcal{M}^*$ are the renormalized variables already defined for nonquantum fluids [8,21].

C. Master and physical singular behavior

Because such transformations of the physical fields in the FT framework have a range of validity including (at least) the first correction-to-scaling [24], our rescaled thermodynamic and correlation functions should conform to the two-term (leading and first-confluent) asymptotic description of singularities within the preasymptotic domain. For example when $\mathcal{T}^*$ goes to zero along the critical isochore, the critical behavior of any rescaled singular property $\mathcal{P}_{\text{qf}}^*$ is given by

$$\mathcal{P}_{\text{qf}}^* = \mathcal{Z}_{\mathcal{P}}^\pm |\mathcal{T}^*|^{\lambda x} [1 + \mathcal{Z}_{\mathcal{P}}^{1,\pm} |\mathcal{T}^*|^\Delta + O(|\mathcal{T}^*|^{2\Delta})], \tag{23}$$

where x (with $\lambda = \pm 1$) and Δ are the associated universal critical exponents [25]. The subscript $+$ is for the homogeneous domain $\mathcal{T}^* > 0$ (i.e., $T > T_c$), and the subscript $-$ is for the nonhomogeneous domain $\mathcal{T}^* < 0$ (i.e., $T < T_c$). The leading amplitudes $\mathcal{Z}_{\mathcal{P}}^\pm$, and the first confluent amplitudes $\mathcal{Z}_{\mathcal{P}}^{1,\pm}$, are master (constant) numbers for all pure fluids. Their respective values (see Sec. III A. below and Table I, columns 3 and 4) are obtained using xenon as a standard critical fluid [21], and using up-to-date estimates [25,24] of universal asymptotic critical quantities (exponents and amplitudes combinations).

When the generalized critical parameters of a pure fluid are known, there is an immediate practical interest to reverse the use of the scale dilatation method. In fact, the basic advantage of this method is its ability to calculate all the amplitudes appearing in the singular divergences expressed at first order of the Wegner expansion in $\Delta \tau^*$. For $\Delta \tau^* \to 0^\pm$, the critical behavior of the physical property P of the selected pure fluid is represented by the two-term equation

$$P = P_0^\pm |\Delta \tau^*|^{\lambda x} [1 + P_1^\pm |\Delta \tau^*|^\Delta + O(|\Delta \tau^*|^{2\Delta})], \tag{24}$$

where $P_0^\pm$ and $P_1^\pm$ are the leading and the first confluent amplitudes. All the values of $P_0^\pm$ and $P_1^\pm$ can then be estimated when the basic set of critical parameters is known for a selected pure fluid, using each unequivocal relation linking the physical quantity to its renormalized one (see Table I, columns 6 and 7).

III. THE HELIUM 3 CASE

A. Notations

The scale dilation method is now applied to the measurements of Zhong *et al.* [14] of the isothermal compressibility (κ_T), the singular specific heat (Δc_V), and the coexisting density difference ($\Delta \rho_{LV} = \rho_L - \rho_V$), along the critical isochore of ^{3}He. We complete these measurements by the estimation of the correlation length (ξ) inferred from the two-scale-factor universality. First, the corresponding notations are introduced for the following.

(i) The physical singular behaviors

$$\xi = \xi_0^\pm |\Delta \tau^*|^{-\nu} [1 + a_\xi^\pm |\Delta \tau^*|^\Delta], \tag{25}$$

$$\chi_{T,\rho}^* = \Gamma^\pm |\Delta \tau^*|^{-\gamma} [1 + a_\chi^\pm |\Delta \tau^*|^\Delta], \tag{26}$$

$$\Delta c_{V,\rho}^* = \frac{A^\pm}{\alpha} |\Delta \tau^*|^{-\alpha} [1 + \alpha a_C^\pm |\Delta \tau^*|^\Delta] + B_{\text{cr}}^*, \tag{27}$$

$$\Delta \tilde{\rho}_{LV} = B |\Delta \tau^*|^\beta [1 + a_M |\Delta \tau^*|^\Delta]. \tag{28}$$

ξ of Eq. (25) is the actual size of the critical fluctuations of the order parameter density. $\chi_{T,\rho}^* = p_c \kappa_T$ of Eq. (26) is the dimensionless form of the susceptibility $\chi_{T,\rho} = \rho^2 \kappa_T$ at $\rho = \rho_c$. $\Delta c_{V,\rho}^* = \rho \Delta c_V \frac{T_c}{p_c}$ of Eq. (27) is the dimensionless form of the singular heat capacity at constant volume. $\Delta \tilde{\rho}_{LV} = \frac{\Delta \rho_{LV}}{2\rho_c}$ of Eq. (28) is the "symmetrical" dimensionless form of the order

TABLE I. Parameters for the master [Eq. (23)] and physical [Eq. (24)] singular behaviors of master ($\mathcal{P}_{qf}^*$) and physical (P) properties, respectively, within the preasymptotic domain, along the critical isochore of the one component fluids. Columns 1–4: Parameters for the master critical behavior of the correlation length [Eq. (29), the susceptibility [Eq. (30)], the specific heat [Eq. (31)], and the order parameter density [Eq. (32)]. Columns 5–7: Calculated parameters for the physical critical behavior of the correlation length [Eq. (25)], the susceptibility [Eq. (26)], the specific heat [Eq. (27)], and the symmetrical density difference [Eq. (28)]. Exponent values (column 2), amplitude ratios values (not explicited here), and $\Delta = 0.50189$, are from central values calculated in Refs. [25,24]. In columns 3 and 4, superscript (asterisk) indicates the leading amplitudes $\mathcal{Z}_\chi^+$ and $\mathcal{Z}_\mathcal{M}$, and the confluent amplitude $\mathcal{Z}_\chi^{1,+}$, which are selected as three independent amplitudes (with their corresponding critical exponents γ, β, and Δ, selected as independent exponents) to characterize the Ising like universal features of the one-component fluid subclass (see text). At each numerical value of the independent amplitude is attached the actual (experimental and theoretical) uncertainty associated to that of the corresponding physical amplitudes Γ^+, B (column 6) and a_χ^+ (column 7) for critical xenon (see text and Ref. [21]). All the numerical values of the other amplitudes are calculated from the theoretical estimation of the critical exponents and amplitude combinations given in Refs. [25,24]. These master numbers have a precision quoted for self-consistency with the central values of universal exponents and amplitude combinations which cannot reflect the actual (experimental and theoretical) uncertainties on the exponent and amplitudes values (see text).

$\mathcal{P}_{qf}^*$	x [25]	$\mathcal{Z}_{\mathcal{P}}^\pm$	$\mathcal{Z}_{\mathcal{P}}^{1,\pm}$	P	$P_0^\pm$	$P_1^\pm$
ℓ_{qf}^*	$\nu = 0.6304$	$\mathcal{Z}_\xi^+ = 0.570365$	$\mathcal{Z}_\xi^{1,+} = 0.37685$	ξ	$\xi_0^+ = \alpha_c \Lambda_{qe}^* (Y_c)^{-\nu} \mathcal{Z}_\xi^+$	$a_\xi^+ = \mathcal{Z}_l^{1,+}(Y_c)^\Delta$
χ_{qf}^*	$\gamma = 1.23959$	$\mathcal{Z}_\chi^+ = 0.119^{(*)}$	$\mathcal{Z}_\chi^{1,+} = 0.555^{(*)}$	$\chi_{T,\rho}^*$	$\Gamma^+ = (\Lambda_{qe}^*)^{d-2}(Z_c)^{-1}(Y_c)^{-\gamma}\mathcal{Z}_\chi^+$	$a_\chi^+ = \mathcal{Z}_\chi^{1,+}(Y_c)^\Delta$
$\Delta \mathcal{C}_{qf}^*$	$\alpha = 0.1088$	$\mathcal{Z}_\mathcal{C}^+ = 0.105656$	$\mathcal{Z}_\mathcal{C}^{1,+} = 0.5231$	$\Delta c_{V,\rho}^*$	$\frac{A^+}{\alpha} = (\Lambda_{qe}^*)^{-d}(Y_c)^{2-\alpha}\mathcal{Z}_\mathcal{C}^+$	$a_\mathcal{C}^+ = \mathcal{Z}_\mathcal{C}^{1,+}(Y_c)^\Delta$
$\mathcal{M}_{qf}^*$	$\beta = 0.325785$	$\mathcal{Z}_\mathcal{M} = 0.468^{(*)}$	$\mathcal{Z}_\mathcal{M}^1 = 0.4995$	$\Delta \widetilde{\rho}_{LV}$	$B = (\Lambda_{qe}^*)^{-1}(Z_c)^{-1/2}(Y_c)^\beta \mathcal{Z}_\mathcal{M}$	$a_M = \mathcal{Z}_\mathcal{M}^1(Y_c)^\Delta$

parameter density in the non homogeneous domain ($T < T_c$). Equations (26)–(28) are written with useful dimensionless variables of fluid related critical phenomena [26] (for which a complementary analysis is made in the next subsection), to be consistent with the normalization of the thermodynamics and the role of the energy and length units given by Eqs. (3) and (4), respectively. The universal values of the critical exponents ν, γ, α, and β, estimated by Guida *et al.* [25] are given in column 2 of Table I. $\Delta = 0.502(\pm 0.004)$ [25] is the lowest value of the confluent exponent. The leading amplitudes $\xi_0^\pm$, $\Gamma^\pm$, $A^\pm$, B, and the confluent amplitudes $a_\xi^\pm$, $a_\chi^\pm$, $a_\mathcal{C}^\pm$, a_M, are fluid dependent quantities; three (two leading amplitudes and one confluent amplitude) of them are characteristic of each one component fluid, in conformity with the two-scale-factor universality.

(ii) The master singular behaviors

$$\ell_{qf}^* = \mathcal{Z}_\xi^+ |\mathcal{T}^*|^{-\nu}[1 + \mathcal{Z}_\xi^+ |\mathcal{T}^*|^\Delta], \qquad (29)$$

$$\chi_{qf}^* = \mathcal{Z}_\chi^\pm |\mathcal{T}^*|^{-\gamma}[1 + \mathcal{Z}_\chi^{1,\pm} |\mathcal{T}^*|^\Delta], \qquad (30)$$

$$\Delta \mathcal{C}_{qf}^* = \mathcal{Z}_\mathcal{C}^\pm |\mathcal{T}^*|^{-\alpha}[1 + \mathcal{Z}_\mathcal{C}^{1,\pm} |\mathcal{T}^*|^\Delta], \qquad (31)$$

$$\mathcal{M}_{qf}^* = \mathcal{Z}_\mathcal{M} |\mathcal{T}^*|^\beta [1 + \mathcal{Z}_\mathcal{M}^1 |\mathcal{T}^*|^\Delta], \qquad (32)$$

where ℓ_{qf}^*, χ_{qf}^*, $\Delta \mathcal{C}_{qf}^*$, and $\mathcal{M}_{qf}^*$ are the renormalized forms of the correlation length, the susceptibility, the singular heat capacity, and the order parameter density, respectively. Note that a master critical constant corresponding to the critical background term of the singular behavior of the specific heat [see Eq. (27)] is ignored in Eq. (31). That infers no limitation on the practical use of this equation since the actual back-

ground constant (to be determined) also contains the contribution of the regular behavior of the specific heat [see Eq. (33) below]. The master (i.e. constant) values of the leading ($\mathcal{Z}_\xi^+$, $\mathcal{Z}_\chi^+$, $\mathcal{Z}_\mathcal{C}^+$, and $\mathcal{Z}_\mathcal{M}$) and confluent ($\mathcal{Z}_\xi^{1,+}$, $\mathcal{Z}_\chi^{1,+}$, $\mathcal{Z}_\mathcal{C}^{1,+}$, and $\mathcal{Z}_\mathcal{M}^1$) amplitudes are given in columns 3 and 4 (respectively), of Table I. As already mentioned in Sec. II C, the values of these master amplitudes are obtained from reference to the asymptotic singular behavior of xenon [8,10,21]. Here, the results are presented in Table I selecting $\mathcal{Z}_\chi^+$, $\mathcal{Z}_\mathcal{M}$, and $\mathcal{Z}_\chi^{1,+}$ as three independent amplitudes to characterize the one-component fluid subclass. Correspondingly, the critical exponents $\gamma = 1.23959$, $\beta = 0.325785$, and $\Delta = 0.50189$, are selected as independent exponents (with these central numerical values estimated in Refs. [25,24]). Then the master values of $\mathcal{Z}_\chi^+$, $\mathcal{Z}_\mathcal{M}$, and $\mathcal{Z}_\chi^{1,+}$ are obtained using the equations given in columns 6 and 7, with $\Gamma^+(\mathrm{Xe}) = 0.0583$, $B(\mathrm{Xe}) = 1.467$, $a_\chi^+(\mathrm{Xe}) = 1.23$, and $Y_c(\mathrm{Xe}) = 4.88$, $Z_c(\mathrm{Xe}) = 0.286$, $\Lambda_{qe}^*(\mathrm{Xe}) = 1$. All the other numbers quoted in Table I are calculated using the self-consistent theoretical estimations of the central values of the universal critical exponents and the universal amplitude combinations given in Refs. [25,24]. As a consequence, their numerical precision cannot reflect the actual (experimental and theoretical) uncertainties. Finally, the main important point to note here is that the numbers quoted in Table I are "fixed" within the same precision level of the universal numbers estimated on the theoretical MR scheme, only using the "best" values of three experimental amplitudes which characterize the Ising-like singular behavior of xenon. In addition, we note that the master correlation length l_{qf}^* of Eq. (29) provides a direct comparison of the size of the critical fluctuations to the range of molecular interaction, in order to control that the basic

condition $l_{qf}^* \gg 1$ is valid for critical phenomena understanding [21]. That provides also a criteria to define the master extension of the Ising-like preasymptotic domain for the one-component fluid subclass.

B. Thermodynamic properties

1. The isothermal susceptibility

Considering a mass unit of the fluid as in the standard thermodynamic presentation of *specific* properties, the susceptibility $\chi_{T,\rho} = \left(\frac{\partial \rho}{\partial \mu_\rho}\right)_T = \rho\left(\frac{\partial \rho}{\partial p}\right)_T \sim [\text{kg}^2\,\text{J}^{-1}\,\text{m}^{-3}]$ is expressed in units of $\frac{\rho_c^2}{p_c}$, while the subscript ρ indicates the thermodynamic normalization per mass unit. Therefore, in Eq. (26), $\chi_{T,\rho}^* = \chi_{T,\rho}\frac{p_c}{\rho_c^2} = \kappa_T\left(\frac{\rho}{\rho_c}\right)^2 p_c = (\tilde{\rho})^2 \kappa_T^*$, with $\kappa_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T$ and $\kappa_T^* = p_c \kappa_T$. $\mu_\rho = \frac{\mu_{\bar{p}}}{m_{\bar{p}}}$ is the chemical potential per mass unit, conjugated from the (mass) density ρ. κ_T is the isothermal compressibility. $\tilde{\rho} = \frac{\rho}{\rho_c}$ is the practical dimensionless form of the density, which differs by a factor Z_c from the dimensionless form $\rho^* = \rho\frac{(\alpha_c)^d}{m_{\bar{p}}}$ obtained with the length unit α_c [7,8]. We note that the above susceptibility $\chi_{T,\rho}$ also differs from the susceptibility $\chi_{T,\bar{p}} = \left(\frac{\partial n}{\partial \mu_{\bar{p}}}\right)_T = n\left(\frac{\partial n}{\partial p}\right)_T \sim [\text{J m}^3]^{-1}$ where the subscript $\bar{p}$ stands for the thermodynamic normalization per particle. Expressing $\chi_{T,\bar{p}}$ in coherent units of $\frac{\beta_c}{(\alpha_c)^d}$ [i.e., using Eqs. (3) and (4)], we obtain $\chi_{T,\bar{p}}^* = \left(\frac{1}{Z_c}\right)^2 \chi_{T,\rho}^* = (n_c^*)^2 \kappa_T^*$ [with $n^* = n(\alpha_c)^d$]. However, pressure $\left(\sim\left[\frac{\text{energy}}{\text{volume}}\right]\right)$ appears appropriately expressed in units of $p_c = \frac{(\beta_c)^{-1}}{(\alpha_c)^d}$, within the both (practical and coherent) dimensionless formulations.

2. The heat capacity at constant volume

The total heat capacity at constant volume $C_V \sim [\text{J K}^{-1}]$ of the fluid mass M is divided by the total fluid volume V, in order to have a unit of $\rho c_{V,\rho}$, where $c_{V,\rho} = \frac{C_V}{M} \sim [\text{J kg}^{-1}\,\text{K}^{-1}]$ is the specific heat at constant volume. The dimensionless specific heat $c_{V,\rho}^*$ is then obtained expressing the total heat capacity in units of $\frac{p_c V}{T_c}$, so that $c_{V,\rho}^* = \rho c_V \frac{T_c}{p_c}$. Therefore, in Eq. (27), the singular specific heat $\Delta c_{V,\rho}^*(\Delta \tau^*)$ is such that the total specific heat $c_{V,\rho}^*(T^*)$ as a function of $T^* = \frac{T}{T_c}$ reads as follows:

$$c_{V,\rho}^*(T^*) = \Delta c_{V,\rho}^*(\Delta \tau^*) + C_B^*(T^*). \tag{33}$$

In Eq. (27), B_{cr}^* is a critical constant while, in Eq. (33), $C_B^*(T^*)$ is the regular background reflecting the analytical part of the free energy. In our coherent formulation of the particle properties, the heat capacity per particle $c_{V,\bar{p}} = \frac{C_V}{N} \sim [\text{J K}^{-1}]$ has the (universal) k_B dimension. As a matter of fact, the heat capacity per particle is the unique measurable thermodynamic property which can be made dimensionless only using the Boltzmann factor k_B, i.e., without reference to α_c and $(\beta_c)^{-1}$. Therefore, when the singular heat capacity at constant volume, normalized per particle, obeys the asymptotic power law

$$\Delta c_{V,\bar{p}} = \frac{A_{0,\bar{p}}^\pm}{\alpha}|\Delta \tau^*|^{-\alpha}[1 + O\{|\Delta \tau^*|^\Delta\}] \tag{34}$$

along the critical isochore, one (+ or −) among the two dimensionless amplitudes $\frac{A_{0,\bar{p}}^\pm}{k_B}$ is mandatorily a characteristic fluid-particle-dependent number (the two amplitudes being related by the universal ratio $\frac{A_{0,\bar{p}}^+}{A_{0,\bar{p}}^-} \cong 0.537$ for $d=3$ [25]). However, hyperscaling features impose that the same length scale is used in thermodynamic and correlation functions. For example, in the case of an "uncompressible" 3D Ising-system of the lattice spacing a_{Ising}, the singular part of the heat capacity normalized by k_B can be expressed in unit of $(a_{\text{Ising}})^d$ [27] [the extensive nature of the total number of particle is then implicitly accounted for in a crystallized solid system since the total volume is proportional to the cell lattice volume filled with a fixed number of particles]. Similarly, in the case of the compressible one-component fluid, one needs to express normalized heat capacity per particle in unit of $(\alpha_c)^d$ (ignoring in this simple dimensional analysis the quantum effects on the microscopic wavelength). The number of particles within the critical interaction cell being $\frac{1}{Z_c}$, we thus define the singular part of the heat capacity for the volume of the critical interaction cell as follows:

$$\Delta c_{V,I}^* = \frac{1}{Z_c}\Delta c_{V,\bar{p}}^*, \tag{35}$$

where $\Delta c_{V,\bar{p}}^* = \frac{\Delta c_{V,\bar{p}}}{k_B}$. Accordingly, $\frac{1}{Z_c}$ takes equivalent microscopic nature of the coordination number in the lattice description of the three dimensional Ising systems, while α_c takes equivalent microscopic nature of the lattice spacing a_{Ising}. Now, for comparison with the notations used in fluid-related critical phenomena where all the thermodynamic potentials are divided by the total fluid volume, we also introduce the heat capacity at constant volume, for a fluid in a container of unit volume, $\Delta c_{V=1} = \frac{\Delta c_{V,\bar{p}}}{v_{\bar{p},c}}$ (labeled here with the subscript $V=1$). Using the above length unit [Eq. (4)], the associated dimensionless form reads

$$\Delta c_{V=1}^* = \frac{\Delta c_{V,\bar{p}}}{k_B} \times \frac{1}{v_{\bar{p},c}(\alpha_c)^{-d}} = \frac{\Delta c_{V=1}}{k_B} \times \frac{1}{(\alpha_c)^{-d}}. \tag{36}$$

Obviously, $\Delta c_{V=1}^*$ is identical to (i) The previous dimensionless form $\Delta c_{V,\bar{p}}^* = \frac{\Delta C_V}{V} \times \frac{T_c}{p_c}$ of the total singular heat capacity $\Delta C_V = N\Delta c_{V,\bar{p}}$ of the constant total fluid volume V, filled with N (fixed) particles and (ii) the dimensionless form $\Delta c_{V,I}^* = \Delta c_{V,\bar{p}}^* \times \frac{1}{Z_c}$ of the singular heat capacity of the microscopic interacting volume $v_{c,I}$ [Eq. (9)], filled with $\frac{1}{Z_c}$ (fixed) particles. In this latter situation, we have an explicit comprehension of the extensive nature of the two independent variables V and N for compressible fluids. Specially, we note here the importance of the thermodynamic normalization for a better understanding of the scaling nature of the critical amplitudes, such as in Eq. (27) for example. Considering the hyperscaling law $2 - \alpha = d\nu$ with its attached universal quantity made by the product

$$(\Delta\tau^*)^2 \times \frac{\Delta c_{V,\rho}^*}{(\alpha_c)^d} \times \xi^d = \text{universal quantity}$$

and rewriting this product such as

$$(\Delta\tau^*)^2 \times \frac{1}{Z_c} \times (\Delta c_{V,\bar{p}}^*) \times \left(\frac{\xi}{\alpha_c}\right)^d = (R_\xi^\pm)^d$$

[with $R_\xi^+ \cong 0.2696$ and $R_\xi^- \cong 0.169$, for $d=3$ [25]], we can easily demonstrate that the universal amplitude combination

$$(R_\xi^\pm)^d = \frac{1}{Z_c}\left(\frac{A_{0,\bar{p}}^\pm}{k_B}\right)\left(\frac{\xi_0^\pm}{\alpha_c}\right)^d \qquad (37)$$

contains the two independent extensive features (volume and number of particles) of the fluid system at the scale of the critical interaction cell. In such a situation, the dimensioned leading amplitudes $A_{0,\bar{p}}^\pm$ (associated to a particle property), and $\xi_0^\pm$ (associated to the microscopic wavelength), have well-understood physical meanings with respect to the universal features of the universality class. As an essential consequence, the universal feature of any singular free energy must then be expressed in terms of the unique remaining energy scale $(\beta_c)^{-1}$. We will return below (see Sec. III C) to this important remark to account for quantum effects in the master singular behavior of the one-component fluid subclass.

3. The densities and the chemical potentials

We finally consider the non homogeneous domain below T_c, where the practical dimensionless form of the symmetrical order parameter density [see Eq. (28)] is defined as

$$\Delta\tilde{\rho}_{LV}(|\Delta\tau^*|) = \frac{\Delta\rho_{LV}(|\Delta\tau^*|)}{2\rho_c} = \frac{\rho_L - \rho_V}{2\rho_c}. \qquad (38)$$

ρ_L (ρ_V) is the liquid (vapor) density of one coexisting phase. Such a dimensionless form comes from the useful variable $\tilde{\rho} = \frac{\rho}{\rho_c}$ (see above), leading to consider the quantity

$$\Delta\tilde{\rho} = \frac{\rho - \rho_c}{\rho_c} = \tilde{\rho} - 1 \qquad (39)$$

as a practical order parameter density, and the quantity

$$\Delta\tilde{\mu}_\rho = \frac{\mu_\rho - \mu_{\rho,c}}{\mu_{\rho,c}} = \tilde{\mu}_\rho - 1 \qquad (40)$$

as a practical ordering field. $\mu_{\rho,c} = \frac{\mu_{\bar{p}}}{m_{\bar{p}}}$ is the specific chemical potential at the critical point. $\tilde{\mu}_\rho = \frac{\mu_\rho}{\mu_{\rho,c}}$ is the practical dimensionless form of the chemical potential, which differs by a factor $(\beta_c\mu_{\bar{p},c})^{-1}$ from our dimensionless form $\mu_{\bar{p}}^* = \beta_c\mu_{\bar{p}}$ obtained with the energy unit $(\beta_c)^{-1}$. From comparison between the two definitions of the order parameter density by Eqs. (22) and (39), we obtain

$$\Delta\tilde{\rho} = Z_c\Delta m^*,$$

$$\Delta\tilde{\rho}_{LV} = Z_c\Delta m_{LV}^*, \qquad (41)$$

where

$$\Delta m_{LV}^* = (n_L - n_V)(\alpha_c)^d. \qquad (42)$$

The main conclusive remark using these conjugated variables $\Delta\tilde{\rho}$ and $\Delta\tilde{\mu}_\rho$, of Eqs. (39) and (40), respectively, concerns the implicit addition of a new length scale factor $\tilde{\alpha}_c = \left(\frac{m_{\bar{p}}}{\rho_c}\right)^{1/d}$ and a new energy scale factor $(\tilde{\beta}_c)^{-1} = m_{\bar{p}}\mu_{\rho,c}$. As a consequence, the nonuniversal nature of each leading amplitude is a complex combination of the critical factors and of the two scale factors associated to universal scaling in fluids.

C. Two-scale-factor universality and quantum effects

In addition to Eqs. (25) to (32), we now introduce the following.

(i) The singular part $\Delta a_\rho(T,\rho) = \frac{\Delta A_\rho}{V}$ of the Helmholtz free energy density, where temperature T and (practical) density $\rho = \frac{Nm_{\bar{p}}}{V}$ are the two selected variables to describe a fluid maintained in a container of constant total volume V. In our case where order parameter density is related to the (natural) number density $n = \frac{N}{V}$, we note $\Delta a(T,n) = \frac{\Delta A}{V}$ this singular part of the Helmholtz free energy density. Due to the appropriate dimensionless form of the pressure mentioned above, both the useful dimensionless form $\Delta a_\rho^*(\Delta\tau^*,\Delta\tilde{\rho}) = \frac{\Delta A_\rho}{V} \times \frac{1}{p_c}$ and the natural dimensionless form $\Delta a^*(\Delta\tau^*,\Delta m^*) = \beta_c\Delta A \times \frac{(\alpha_c)^d}{V}$ are identical, except for the use of two distinct reduced forms $\Delta\tilde{\rho}$ and Δm^* of the order parameter density. Along the critical isochore $\Delta\tilde{\rho} = \Delta m^* = 0$, the singular part of the free energy behaves as

$$\Delta a_\rho^*(\Delta\tau^*) \equiv \Delta a^*(\Delta\tau^*) = A'^\pm|\Delta\tau^*|^{2-\alpha}[1 + O\{|\Delta\tau^*|^\Delta\}]. \qquad (43)$$

The basic thermodynamic definitions of the physical properties are $\Delta\tilde{\mu}_\rho(\Delta\tau^*,\Delta\tilde{\rho}) = \left(\frac{\partial\Delta a_\rho^*}{\partial\Delta\tilde{\rho}}\right)_{\Delta\tau^*}$, or $\Delta\mu_{\bar{p}}^*(\Delta\tau^*,\Delta m^*) = \left(\frac{\partial\Delta a^*}{\partial\Delta m^*}\right)_{\Delta\tau^*}$; $\chi_{T,\rho}^*(\Delta\tau^*,\Delta\tilde{\rho}) = \left(\frac{\partial\Delta\tilde{\rho}}{\partial\Delta\tilde{\mu}_\rho}\right)_{\Delta\tau^*}$, or $\chi_{T,\bar{p}}^*(\Delta\tau^*,\Delta m^*) = \left(\frac{\partial\Delta m^*}{\partial\Delta\mu_{\bar{p}}^*}\right)_{\Delta\tau^*}$; and $\frac{\Delta c_{V,\rho}^*(\Delta\tau^*,\Delta\tilde{\rho})}{T^*} = -\left(\frac{\partial^2\Delta\tilde{a}_\rho}{\partial(T^*)^2}\right)_{\Delta\tilde{\rho}}$, or $\frac{\Delta c_V^*(\Delta\tau^*,\Delta m^*)}{T^*} = -\left(\frac{\partial^2\Delta a^*}{\partial(T^*)^2}\right)_{\Delta\tilde{\rho}}$.

(ii) The renormalized singular free energy density $\mathcal{A}_{qf}^*(\mathcal{T}^*,\mathcal{M}_{qf}^*)$ which, along the isocline $\mathcal{M}_{qf}^* = 0$, asymptotically behaves as

$$\mathcal{A}_{qf}^*(\mathcal{T}^*) = \mathcal{Z}_\mathcal{A}^\pm(\mathcal{T}^*)^{2-\alpha}[1 + O\{(\mathcal{T}^*)^\Delta\}] \qquad (44)$$

with respect to the renormalized thermal field $\mathcal{T}^*$ going to zero. Correspondingly, the thermodynamic definitions of the renormalized properties of present interest are $\mathcal{H}_{qf}^*(\mathcal{T}^*,\mathcal{M}_{qf}^*) = \left(\frac{\partial\mathcal{A}_{qf}^*}{\partial\mathcal{M}_{qf}^*}\right)_{\mathcal{T}^*}$, $\chi_{qf}^*(\mathcal{T}^*,\mathcal{M}_{qf}^*) = \left(\frac{\partial\mathcal{M}_{qf}^*}{\partial\mathcal{H}_{qf}^*}\right)_{\mathcal{T}^*}$, $\frac{\Delta c_{qf}^*(\mathcal{T}^*,\mathcal{M}_{qf}^*)}{\mathcal{T}^*} = -\left(\frac{\partial^2\mathcal{A}_{qf}^*}{\partial\mathcal{T}^{*2}}\right)_{\mathcal{M}_{qf}^*=0}$ [with $\mathcal{Z}_\mathcal{A}^\pm = \frac{\mathcal{Z}_C^\pm}{\alpha(1-\alpha)(2-\alpha)}$].

It is thus easy to obtain the relations reported in columns 6 and 7 of Table I, using the above basic thermodynamic definitions of the renormalized and physical variables. That also provides a comprehensive understanding of the quantum effect correction to master singular behavior.

As a matter of fact, following the argument first proposed by Widom [28], the renormalized energy associated with the
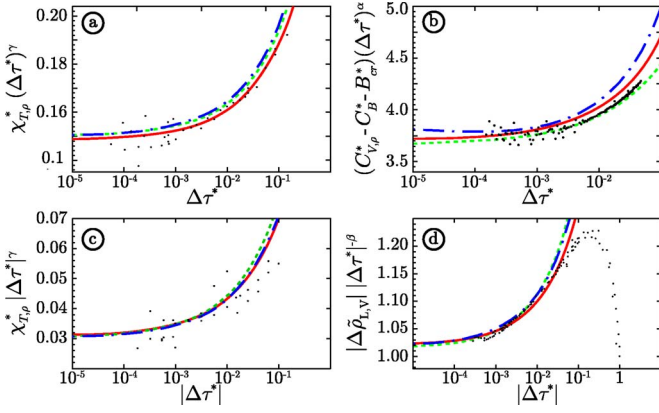
FIG. 1. (Color online) Asymptotic two-term predictions compared to the ^{3}He measurements (black points). The solid (red) lines are the actual predictions by the dilated scale method [see Eq. (24) and column 8 of Table II]. The dot-dashed (blue) lines are the two-term prediction obtained from the best fit by the MSR $\Phi^4_{d=3}(1)$ model [14] [Eq. (24); column 2 of Table II]. The dashed (green) lines are the two-term prediction obtained from CPM model [36] [Eq. (24); column 6 of Table II]. (a) Susceptibility measurements for $T > T_c$ (corresponding to upper part of Fig. 1 in Ref. [14]). (b) Specific heat measurements for $T > T_c$, where the small difference in the additional constant term $C_B^* + B_{cr}^*$ is accounted in the vertical scale (see also the lower part of Fig. 3 in Ref. [14]) (c) Susceptibility measurements for $T < T_c$ (see also the lower part of Fig. 1 in Ref. [14]). (d) Liquid-gas coexisting density measurements (see also Fig. 4 in Ref. [14]).

spontaneous density fluctuations that extend over a distance ℓ_{qf}^* must be of order $(\beta_c)^{-1}$, leading to a renormalized free energy density of order $[\beta_c(\alpha_c)^d]^{-1}$. Along the critical isochore, this energy will be associated to $\mathcal{A}_{qf}^*(\mathcal{T}^*)$ of Eq. (44). The product $\mathcal{A}_{qf}^*(\mathcal{T}^*) \times (\ell_{qf}^*)^d$ being a universal quantity, the relative quantum correction to the renormalized singular free energy reads

$$\mathcal{A}_{qf}^*(\mathcal{T}^*) = (\Lambda_{qe}^*)^d \mathcal{A}^*(\mathcal{T}^*) \tag{45}$$

due to the Eq. (15) for ℓ_{qf}^*. $\mathcal{A}^*(\mathcal{T}^*)$ must be the renormalized singular free energy already defined for nonquantum fluids such as

$$\mathcal{A}^* = \beta_c(\alpha_c)^d \times \frac{\Delta A}{V} \tag{46}$$

Therefore, from the comparison between the leading terms of the renormalized and the physical second derivatives of the singular free energy densities with respect to their associated thermal fields, we obtain

$$A^{\pm} = \frac{1}{Z_c} \frac{A_{0,\bar{p}}^{\pm}}{k_B} = (\Lambda_{qe}^*)^{-d}(Y_c)^{2-\alpha} \mathcal{Z}_{\mathcal{C}}^{\pm}. \tag{47}$$

In addition to the explicit Y_c and Λ_{qe}^* dependences of the leading dimensionless amplitude $A^{\pm}$, the above Eq. (47) also shows the role of the particle number $\frac{1}{Z_c}$ as the multiplicative factor to the particle leading amplitude $A_{0,\bar{p}}^{\pm} \sim [k_B]$. That provides understanding of the master (i.e., unique) singular be-

haviors of the one-component fluid subclass in terms of the master (i.e., constant) properties of the critical interaction cell of any one-component fluid. Similarly, from the comparison between the leading terms of the renormalized and the physical correlation lengths, we obtain

$$\xi_0^{\pm} = \alpha_c \Lambda_{qe}^*(Y_c)^{-\nu} \mathcal{Z}_{\xi}^{\pm}. \tag{48}$$

In Eqs. (47) and (48), $A_{0,\bar{p}}^{\pm} \sim [k_B]$ and $\xi_0^{\pm} \sim [\text{length}]$ have the appropriate $Q_c^{\min}$ and Λ_{qe}^* dependences to satisfy the universal amplitude combination of Eq. (37). These two Eqs. (47) and (48), or more generally, all the relations given in the column 6 of Table I, also demonstrate that the estimation of the adjustable parameter $\lambda_{q,f}$, introduced throughout the Eqs. (12) and (13), is unequivocally made from the leading power law behavior of any property, when $Q_c^{\min}$ is known. That provides a very sensitive test of the above phenomenological approach to account for quantum effects, provided that the same length scale α_c and the same energy scale $(\beta_c)^{-1}$ are used for thermodynamic and correlation functions at $T \cong T_c$. In such a coherent thermodynamic normalization scheme, the relative quantum modification [proportional to $(\Lambda_{qe}^*)^d$] of the energy within the critical interaction cell is correlated to the relative quantum modification of the microscopic wave number [proportional to Λ_{qe}^*]. We thus provide the microscopic quantum mechanical modification which complements the macroscopic argument of Widom [28] and Staufer *et al.*'s [29] that the free energy associated to fluctuations of size ξ were solely responsible for the singular contribution of thermodynamic potentials and correlation functions.

D. ^{3}He results

For the fermionic quantum fluid ^{3}He, the $Q_c^{\min}$ set is composed of the following critical coordinates $T_c = 3.315546$ K, $p_c = 1.14724 \; 10^5$ Pa, $\rho_c = 41.45$ kg m^{-3}, and $\gamma_c' = 1.1759 \; 10^5$ Pa K^{-1} [14]. Using Eqs. (4)–(6), the values of the four scale factors are $(\beta_c)^{-1} = 4.5776 \; 10^{-23}$ J, $\alpha_c = 7.362 \; 10^{-10}$ m, $Y_c = 2.39837$, $Z_c = 0.301284$. By χ^2 optimization only using the susceptibility data above and below T_c in the range $|\Delta \tau^*| < 510^{-3}$, with $\frac{\Gamma^+}{\Gamma^-} = \frac{\mathcal{Z}_{\chi}^+}{\mathcal{Z}_{\chi}^-} = 4.79$ [25], the adjustable parameter $\lambda_{q,f}$ takes the numerical value $\lambda_{q,^3\text{He}} = 0.146423$, leading to $\Lambda_{qe}^* = 1.11009$. For the specific case of the heat capacity, the additional critical (B_{cr}^*) and background (C_B^*) terms are treated as one single adjustable constant ($B_{cr}^* + C_B^*$).

The main results are illustrated in Fig. 1 where the comparison is made with the recent published experimental data (black points in Fig. 1) of Zhong *et al.* [14]. In order to simplify the comparison, the same corresponding scaled data by the asymptotic power law term $(\Delta \tau^*)^{-x}$ were used for susceptibility and heat capacity above T_c, and for coexisting liquid vapor densities below T_c, which improves the sensitivity of the relative representation from the asymptotic amplitude values. Obviously, that provides a simultaneous test of the quantum effect contribution since, among the four leading amplitudes Γ^+, A^+, Γ^-, and B, only one is readily

TABLE II. Calculated values of the asymptotic critical amplitudes, from fitting the ^{3}He results [14] by application of the crossover functions obtained on the minimal substraction renormalization (MSR) scheme (columns 2 [14] and 3 [30]), the massive renormalization (MR) scheme (columns 4 and 5 [30]), and the crossover Landau model (CPM) of the equation of state (columns 6 [36] and 7 [30]), and using the equations of Table I obtained from the scale dilation method (column 8), with $T_c = 3.315546$ K, $p_c = 1.14724 \ 10^5$ Pa, $\rho_c = 41.45$ kg m^{-3}, $\gamma'_c = 1.1759 \ 10^5$ Pa K^{-1}, [14] and $\lambda_{q,^3He} = 0.146423$ (see text). Using Eqs. (4) to (6) and the definition of Λ^*_{qe} from Eqs. (12)–(14), the values of the (five) characteristics parameters for ^{3}He are $(\beta_c)^{-1} = 4.5776 \ 10^{-23}$ J, $\alpha_c = 7.362 \ 10^{-10}$ m, $Y_c = 2.39837$, $Z_c = 0.301284$, and $\Lambda^*_{qe} = 1.11009$. Note that the numbers in column 8 are quoted for a numerical precision which do not reflect their attached level of the (experimental and theoretical) uncertainties (see text for details).

Amplitude	MSR [14]	MSR [30]	MR6 [30]	MR7 [30]	CPM [36]	CPM [30]	This work
ξ^+_0(Å)	2.71±0.02				2.68±0.04		2.68541
a^+_ξ	0.732±0.007						0.58474
Γ^+	0.150±0.007	0.147±0.001	0.146±0.001	0.148±0.001	0.150±0.002	0.153±0.001	0.148247
Γ^-	0.0303±0.0015	0.0299±0.0003	0.0308±0.0001	0.0310±0.0001		0.0310±0.0002	0.030953
a^+_χ	0.98±0.08	1.10±0.01	1.13±0.01	1.17±0.01	0.941±0.007	0.81±0.01	0.860931
a^-_χ	4.29±0.34	4.83±0.05	3.58±0.05	5.30±0.07		4.17±0.07	4.01366
$\frac{A^+}{\alpha}$	3.73±0.45	3.76±0.05	3.72±0.01	3.84±0.02	3.548±0.031	3.63±0.02	3.71132
$\frac{A^-}{\alpha}$	6.97±0.83	7.03±0.10	6.883±0.026	7.149±0.027	6.823±0.01	6.935±0.04	6.90948
αa^+_C	1.2±0.1	0.99±0.01	1.13±0.01	1.07±0.01	0.712±0.006	0.61±0.01	0.810892
αa^-_C	1.1±0.1	0.92±0.01	1.17±0.01	0.83±0.01	0.593±0.012	0.74±0.01	0.59712
$B^*_{cr}+C^*_B$	−1.65±0.85	−1.67±0.13	−1.64±0.04	−1.81±0.04	−0.96±1.0	−1.23±0.05	−1.40
B	1.020±0.006	1.021±0.003	1.008±0.004	1.039±0.004	1.0047	1.028±0.004	1.02134
a_M	0.91±0.02	0.91±0.01	1.001±0.023	0.218±0.003	0.8441	0.73±0.01	0.77484

sufficient to define the unequivocal Λ^*_{qe} dependence. Moreover, to illustrate the first confluent term contribution associated to the scale dilatation method, the full (red) lines and the dot-dashed (blue) lines in Fig. 1 correspond to the respective first-order Wegner expansions obtained from Table I, and from the initial fit of Zhong et al. [14], using the minimal substraction renormalization (MSR) scheme (see also the corresponding numerical values of the amplitudes listed in Table II). For the three selected properties, the predicted singular behavior fits well the experimental results and matches the theoretical predictions of the minimal-substraction renormalization scheme.

More generally, as shown in Table II, the two-term asymptotic results obtained with the scale dilatation method are in good agreement with the two-term parametric modeling results recently obtained by Zhong and Barmatz [30], based on three different theoretical models. Two of these models come from the two main field-theoretical renormalization schemes that treat classical-to-critical crossover phenomena, namely the minimal-substraction renormalization scheme of Dombs and co-workers [31–35], and the massive renormalization scheme of Bagnuls and Bervillier [4,5,9,24]. These schemes are only applied to the primary critical path corresponding to the homogeneous and nonhomogeneous domain along the critical isochore. The third model, namely the crossover parametric model, proposed by Agayan and co-workers [36,37], is a complete parametric equation of state developed from a phenomenological crossover transformation of a classical Landau expansion of the singular free energy [38–40]. Although it is phenomenological, this crossover Landau model was successfully applied to several one-component fluids. A previous comparison of the results obtained by the crossover Landau model and the scale

dilatation method was already made in the case of seven nonquantum fluids [21]. In Table II are reported:

(i) Columns 2 and 3 labeled MSR, the results obtained by Zhong et al. [14] and Zhong and Barmatz [30] from the minimal-substraction renormalization scheme;

(ii) Columns 4 and 5 labeled MR6 and MR7, the results obtained by Zhong and Barmatz [30] from the massive renormalization scheme in the sixth- [5,9] and seventh-loop [24] series;

(iii) Columns 6 and 7 labeled CPM, the results obtained by Agayan et al. [36] and Zhong and Barmatz [30] from the crossover parametric model;

(iv) Column 8, the results obtained in this work applying the scale dilatation method, which provides the equations given in Table I. We recall that the numbers quoted in this column only account for numerical precision of the theoretical estimations of the central values of the critical exponents and amplitude combinations given in Refs. [25,24].

The excellent agreement between the amplitude values permits to discuss now the introduction of the adjustable parameters in the modeling, and to explain why only two adjustable parameters in the models are significant with respect to the fit quality, as concluded by Zhong and Barmatz [30].

IV. ^{3}He CRITICAL MODELING

A. The two renormalization schemes along the critical isochore

As clearly mentioned in the Appendix D of Ref. [14], the three free parameters of the MSR-model originate from the undetermined integration constants z_ϕ, z_a, and z_μ, associated

to the flow equations of their respective $Z_\phi(u)$, $Z_r(u)$, and $Z_u(u)[Z_\phi(u)]^{-2}$ field theoretical functions (here we have adopted the Zhong *et al.* notation for z_a and z_μ, adding z_ϕ as being the undetermined integration constant to solve Eq. (7) of Ref. [14]). These integration constants are system dependent and can be obtained by fitting experimental data to the theory. However, considering uniquely the critical isochore, the given set composed by the explicit adjustable parameters (such as $\{u,\mu,a\}$ in Refs. [13,14]), or calculated parameters (such as the leading amplitudes and t_0 in Refs. [13,14]), results in complicated scaled forms of combinations between z_ϕ, z_a, and z_μ. Specifically, to correctly account for the z_ϕ system dependence, one needs to use several properties. The susceptibility fitting results reported in Fig. 2 of Ref. [14], where only two (μ and t_0) among the three scaled parameters (μ, a and t_0) have the expected power law dependence on $1-\frac{u}{u^*}$ near the fixed point ($u=u^*$), should also be due to a non-representative test of one asymptotical scaled form. A preliminary comparison of the functional forms of the leading amplitudes obtained from the MSR model and the scale dilatation method for the case of the non-quantum fluid subclass, suggests, for example, that the true independent scaled factors of each physical system are then such as

$$\frac{z_a}{(z_\mu)^{\zeta_r^*}} \propto Y_c \tag{49}$$

and

$$\frac{z_\phi}{(z_\mu)^{\zeta_\phi^*}} \propto Z_c. \tag{50}$$

In Eqs. (49) and (50), $\zeta_r^*=\zeta_r(u^*)=1-\frac{1}{\nu}$ and $\zeta_\phi^*=\zeta_\phi(u^*)=-\eta$ are the respective values of the field theoretical functions at the Ising fixed point $u=u^*$ (see the notations of Zhong *et al.*'s [14]). This suggestion should be useful for a possible rescaling of the leading amplitudes which gives better evidence for the two asymptotical parameters which are readily independent in the modeling form of the minimal-substraction renormalization scheme.

The two-term master asymptotical behavior obtained from the scale dilatation method can be described [41] by the massive renormalization scheme of Bagnuls and Bervillier, thanks to its formal analogy to the basic analytical hypotheses of the renormalization [22,23]. Using a similar approach which introduces one common (i.e., P^* independent) crossover parameter ϑ_{3He}, and adjustable prefactors $\mathbb{P}_{0,^3He}^\pm$ for each dimensionless property P^*, we obtain the following values $\mathbb{L}_{0,^3He}^+=1.2925$, $\mathbb{X}_{0,^3He}^+=1.818$, $\mathbb{C}_{0,^3He}^+=2.1503$, and $\mathbb{M}_{0,^3He}^\pm=1.0894$, for the leading prefactors of the correlation length, the susceptibility, the heat capacity and the coexistence curve, respectively. These four leading parameters are interrelated by the following combinations $\mathbb{L}_{0,^3He}^+(\mathbb{C}_{0,^3He}^+)^{1/d}$ $=1$ and $\frac{\mathbb{X}_{0,^3He}^+}{(\mathbb{M}_{0,^3He}^\pm)^2}(\mathbb{L}_{0,^3He}^+)^{-d}=1$, so that only two of them are independent, by virtue of the two scale factor universality. The estimated value of the crossover parameter is $\vartheta_{3He}=0.0113$. The mean crossover functions [42] will be used in

a future work to implement the master estimation of their free parameters from the four scale factors defined by $Q_{c,a_{\bar{p}}}^{\min}$.

B. The crossover parametric model of the EOS

The crossover parametric model comes from the crossover Landau model (CLM) of the EOS, based on a phenomenological transformation of a classical Landau expansion of the singular free energy of the fluid as a function of the local order parameter density. In such a model, the simplest crossover description involves three free parameters, made of the two coupling constants a_0 and u_0 and one gradient prefactor c_0 (see, for example, Ref. [36] for notations). After transformation of variables and coefficients, the three initial system-dependent coefficients a_0, u_0, and c_0, are replaced by two dimensionless asymptotic scaling parameters (noted c_t and c_ρ in the general CLM approach) and one dimensionless crossover parameter (noted g). However, from the field theory framework, any description of a 3D Ising-like system with "finite" cutoff, needs to maintain the appropriate interdependence between the nonuniversal parameters, specially the microscopic wavelength $\Lambda_0\sim[\text{length}]^{-1}$ and the coupling constant $u_0\sim[\text{length}]$. Introducing then a common *arbitrary* length scale unit permits one to replace the product $u_0\Lambda_0$ by the product $u\Lambda$ of the corresponding dimensionless wavelength Λ and dimensionless coupling constant u. The convenient normalization $\bar{u}=\frac{u}{u^*}$, where u^* corresponds to the universal value at the non-Gaussian fixed point, leads to an arbitrary choice for the dimensionless microscopic wavelength Λ and the normalized coupling parameter $\bar{u}$, provided that $\bar{u}\Lambda$ remains finite in order to account for theoretical infinite cutoff approximation $\Lambda\to\infty$ and $\bar{u}\to0$. In this infinite-cutoff limit where g is related to the Ginzburg number [40], the crossover behavior is then universal by rescaling the thermal-field-like variable using a single crossover parameter (such as $g=\frac{(\bar{u}\Lambda)^2}{c_t}=\Delta\tau_X^*$, or such as the crossover temperature $t_X=c_t\Delta\tau_X^*$, equivalently [40]). However, at the general symmetrical fourth-order (with only two independent coupling quantities a_0 and u_0) of the phenomenological transformation of the classical Landau expansion of the singular free energy, the crossover behavior is governed by the two dimensionless parameters g and $\bar{u}$. In such a situation, all the dimensionless quantities are canonical constants, provided one has defined a single microscopic characteristic length scale for each fluid. That provides implicit connection between Λ and $\bar{u}$, or equivalently between Λ and, for example c_t, when the explicit $g=\frac{(\bar{u}\Lambda)^2}{c_t}$ dependence is accounted for, as mentioned above. As a consequence, the only way to monitor the asymptotic critical behavior of the crossover Landau model is to change $\bar{u}$, or equivalently c_t. We recall that, in a previous analysis of the corresponding results for the case of seven nonquantum fluids [21], we have shown that

$$c_t(\bar{u}\Lambda) = Y_c \times f_t(\bar{u}\Lambda) \tag{51}$$

and

$$c_\rho(\bar{u}\Lambda) = (Z_c)^{1/2} \times f_\rho(\bar{u}\Lambda) \qquad (52)$$

are unequivocally related to our scale factors Y_c and Z_c, respectively. In Eqs. (51) and (52), $f_t(\bar{u}\Lambda)$ and $f_\rho(\bar{u}\Lambda)$ are two appropriate universal power laws of the product $\bar{u}\Lambda$, uniquely. In the following, we will also provide one possible estimation of the coupling constants $a_0(g)$ and $u_0(g)$ from Y_c and Z_c, using the system-dependent coefficients of the crossover parametric model.

The three-parameter crossover parametric model contains two asymptotic scaling parameters, noted l_0 and m_0, and again the crossover parameter g. A comparison between definitions of asymptotic amplitudes Γ^+ and B leads to the following relations:

$$l_0 = \frac{3.38317}{\mathcal{Z}_\chi} \frac{\mathcal{Z}_\mathcal{M}}{3.28613}(\Lambda_{qe}^*)^{-2}(Z_c)^{1/2}(Y_c)^{\beta+\gamma} \qquad (53)$$

and

$$m_0 = \frac{\mathcal{Z}_\mathcal{M}}{3.28613}(\Lambda_{qe}^*)^{-1}(Z_c)^{-1/2}(Y_c)^\beta \qquad (54)$$

(see our Table I and Table III of Ref. [36] for details). Our direct estimation of the two free values $l_0=7.0929$ and $m_0=0.3108$ from Eqs. (53) and (54), are in close agreement with the values $l_0=6.89\pm0.12$ and $m_0=0.306\pm0.01$, deduced from the fitting procedure of Agayan et al. (see Ref. [36]), and with the values $l_0=6.902\pm0.012$ and $m_0=0.3128\pm0.0004$, recently obtained by Zhong and Barmatz [30] in their comparison of theoretical models of crossover behavior. Moreover, as previously mentioned, from the identification of the leading amplitudes given in Table I of Ref. [36], calculated using, either the crossover Landau model, or the crossover parametric model, it is now easy to show that the two coupling constants a_0 and u_0 are related to Y_c and Z_c (and Λ_{qe}^*, obviously), by the following relations:

$$a_0(g) = (\Lambda_{qe}^*)^{-1} Z_c (Y_c)^\gamma f_{a_0}(g) \qquad (55)$$

and

$$u_0(g) = \Lambda_{qe}^*(Z_c)^2(Y_c)^{2\beta-\gamma} f_{u_0}(g). \qquad (56)$$

In Eqs. (55) and (56), $f_{a_0}(g)$ and $f_{u_0}(g)$ are two appropriate universal power laws of the crossover parameter g.

The first confluent amplitude for the susceptibility obtained from the crossover parametric model reads $a_\chi^+ = g_\chi^+ g^{-\Delta_s}(1-\bar{u})$, with $g_\chi^+=0.590$, $\Delta_s=0.51$, and $g=\frac{(\bar{u}\Lambda)^2}{c_t}$ (see Table III of Ref. [36], with $a_\chi^+ \equiv \Gamma_1^+$ in the notations of Agayan et al.). The identification with our corresponding amplitude $a_\chi^+ = \mathcal{Z}_\chi^{1,+}(Y_c)^\Delta$ (Table I), gives

$$g_\chi^+\left(\frac{\bar{u}\Lambda}{(c_t)^{1/2}}\right)^{-2\Delta_s}(1-\bar{u}) = \mathcal{Z}_\chi^{1,+}(Y_c)^\Delta \qquad (57)$$

demonstrating the unequivocal relation between $g^{1/2}=\frac{\bar{u}\Lambda}{(c_t)^{1/2}}$ and Y_c. However, the rescaled coupling constant $\bar{u}$ remains dependent, on the one hand, to the correlation between the

three adjustable dimensionless parameters c_t, $\bar{u}$, and Λ of the model, and on the other hand, to the master value $\mathcal{Z}_\chi^{1,+}=0.555$ initially estimated from the analysis of the isothermal compressibility data of xenon. That implies the implicit introduction of one characteristic microscopic length which must have unique "thermodynamic" definition [by Eq. (4)], whatever the selected one-component fluid. In that "normalized" situation, our present value $a_\chi^+=0.861$ for ^{3}He, is in good agreement with for example the values $a_\chi^+=0.941\pm0.007$ [36] and $a_\chi^+=0.81\pm0.01$ [30] obtained from data fitting with the crossover parametric model (see below for more details on the uncertainty associated to the a_χ^+ determination). Accounting for the arbitrary relation $\frac{\Lambda}{(c_t)^{1/2}}=\pi$ adopted by the authors in Ref. [36], our calculated value $\bar{u}=0.18075$ from Eq. (57), yields $g^{1/2}=0.5678$, which compares favourably to $g^{1/2}=0.528\pm0.003$ obtained from the data fitting performed by Agayan et al. [36]. Accounting for the arbitrary relation $\frac{\Lambda}{(c_t)^{1/2}}=\frac{\pi}{\sqrt{6}}$ adopted by the authors in Ref. [30], with Λ fixed at unity (yielding to $g^{1/2}=u^*=0.472$), our calculated value $\bar{u}=0.35187$ from Eq. (57) (yielding $g^{1/2}=0.4513$), compares favorably to $\bar{u}=0.368\pm0.004$ obtained from the data fitting of Zhong et al. [30]. The $\sim 10\%$ residual difference between these two estimates of the fluid-dependent parameters, reflects the small differences between theoretical values of universal exponents and amplitude combinations, added to the uncertainty in the direct estimation of the confluent amplitude, the latter one being greater than 10%. For example, using "equivalent" crossover Landau modeling of the same ^{3}He experimental data, the resulting values are $a_\chi^+=0.946\pm0.006$ and $a_\chi^+=1.000\pm0.028$ in Ref. [37], $a_\chi^+=0.941\pm0.007$ in Ref. [36], and $a_\chi^+=0.81\pm0.01$ in Ref. [30], while using the minimal-substraction renormalization scheme, the resulting values are $a_\chi^+=1.01\pm0.08$ in Ref. [13], $a_\chi^+=0.98\pm0.08$ or $a_\chi^+=1.13\pm0.01$ in Ref. [14], and $a_\chi^+=1.10\pm0.01$ in Ref. [30], leading to the practical "mean" value $a_\chi^+=0.97\pm0.16$ (see also Table II). Nevertheless, this agreement confirms our previous analyses [10,21] of the confluent correction to scaling for the one-component fluid subclass satisfying the classical-to-critical crossover description along the ideal RG trajectory [43,44].

V. CONCLUSIONS

The present study in terms of the dilated physical fields for quantum fluids adds only one well-defined adjustable parameter, which accounts for microscopic quantum effects only asymptotically close to the critical point ($T \cong T_c$). The adjustable parameter is introduced in a phenomenological manner which maintains universal feature of the singular free energy in a appropriate microscopic volume. Since our selected standard fluid is xenon, we provide here a complementary new light to the recent discussions [13,45] about the definitions of the crossover temperature t_X [related to the crossover parameter g, (or the Ginzburg number), as mentioned in Sec. IV B]. As an essential new consequence, we note that $t_X \propto \frac{1}{Y_c}$ along the critical isochore, for $T>T_c$. Therefore, our two-term asymptotic hyperscaling seems also compatible with (at least) the first-order contribution to the criti-

cal crossover. However, this observed supplementary constraint is not a necessity from the field theory framework [24]. Consequently, our next work [46] is to provide thermodynamic fundaments for the asymptotic master behavior of thermodynamic and correlation functions which was inferred from the above minimal information. We also propose a convenient mean form [41,42] of the max and min forms for each complete crossover function recently derived by Bagnuls and Bervillier [24]. Such mean functions can be appropriately modified to account for the results obtained by the scale dilatation method [41], extending thus the analysis of the crossover behavior of the one-component fluids outside their Ising-like preasymptotic domain.

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[11] The subscript $a_{\bar{p}}$ in $Q_{c,a_{\bar{p}}}^{\min}$ notation, recalls for the Helmholtz free energy per particle $a_{\bar{p}}(T,v_{\bar{p}}) = \frac{A(T,V/N,1)}{N}$, where $A(T,V,N)$ is the total Helmholtz free energy of the fluid and T (temperature), V (total volume), N (total number of particles), its three natural variables. $v_{\bar{p}} = \frac{V}{N}$ is the particle volume. From the general point of view of the thermodynamics, the equilibrium states of a one-component system at constant amount of matter N, are entirely defined by the knowledge of the normalized thermodynamic potential $a_{\bar{p}}(T,v_{\bar{p}})$ or, alternatively but equivalenty, by the knowledge of the two equations of state $s_{\bar{p}}(T,v_{\bar{p}}) = \left(\frac{\partial a_{\bar{p}}}{\partial T}\right)_{v_{\bar{p}}}$ and $p(T,v_{\bar{p}}) = \left(\frac{\partial a_{\bar{p}}}{\partial v_{\bar{p}}}\right)_T$. $s_{\bar{p}}$ (conjugated to T) is the entropy of the particle and p (conjugated to $v_{\bar{p}}$) is the pressure. These equilibrium states are represented by one 3D characteristic surface of equation $\Phi_{a_{\bar{p}}}(a_{\bar{p}},T,v_{\bar{p}}) = 0$ (using the two natural variables and the corresponding normalized thermodynamic potential), or alternatively but equivalently, by two 3D phase surfaces of equations $\Phi_{a_{\bar{p}}}^{s_{\bar{p}}}(s_{\bar{p}},T,v_{\bar{p}}) = 0$ and $\Phi_{a_{\bar{p}}}^{p}(p,v_{\bar{p}},T) = 0$, (using then the two natural variables and one conjugated variable), respectively. Only the latter phase surface can be constructed from p,V,T,M (total mass) measurements, when the mass $m_{\bar{p}}$ of the particle is known [with $M = Nm_{\bar{p}}$].

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